NAN-003-001617 Seat No. _______

B. Sc. (Sem. VI) (CBCS) Examination

March / April - 2017

BSMT-602(A) - Mathematics

(Analysis-II & Abstract Algebra-II)

Faculty Code : 003
Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory
(2) Give calculation where it is expected.

1 Answer the followings : **20**

(1) Define: Open Cover.

(2) Define: Compact Set.

(3) Define: Bounded Set.

(4) Define: Interval.

(5) Give an example of a subset of the metric space $\mathbb{R}$ which is neither open nor closed nor compact nor connected.

(6) Is $\mathbb{R}$ a compact set ? Justify.

(7) Define : Sequentially Compact Metric Space.

(8) Define : Laplace Transform.

(9) L{sinh(at)} = _________.

(10) $L^{-1}\left(\frac{1}{s-a}\right) =$ _________.

(11) Define: Homomorphism of groups.

(12) Define: Natural Mapping.

(13) Define: Kernel of Homomorphism of groups.

(14) Define: Unit element in a ring.

(15) Define: Principal Ideal Ring.

(16) Define: Polynomial.

(17) Define: Degree of a polynomial.

(18) Define: Monic Polynomial.

(19) Define: Reducible Polynomial.

(20) Define: GCD of polynomials.

2 (a) Answer any **Three** : **6**

(1) Show that $\mathbb{Q} \subset \mathbb{R}$ is not compact in $(\mathbb{R}, u)$.

(2) Show that [1,5) and (5,7) are separated sets of the metric space $(\mathbb{R}, u)$.

(3) Show that $\mathbb{R} - \{1,2,3\}$ is not a connected set.

(4) Find $L(e^{at} t \sin(bt))$.

(5) Find $L\{\sin(2t)\ \sin(3t)\}$.

(6) Find inverse Laplace transform of $\frac{s+2}{s^2 - 4s + 13}$.

(b) Answer any **Three** : **9**

(1) Prove that a non-empty subset of the discrete metric space is connected if and only if it is singleton.

(2) Prove that a closed interval of the metric space $(\mathbb{R}, u)$ is always a closed set.

(3) Prove that the intersection of two compact sets is again a compact set in a metric space.

(4) Find $L\{(\sin t - \cos t)^2\}$.

(5) State and prove the change of scale property of Laplace transform.

(6) Find inverse Laplace transform of $\frac{s+1}{s^2(s^2+1)}$.

(c) Answer any **Two** : **10**

(1) Let (X, d) be a metric space and let E_1 & E_2 be any two connected subsets of X. Then prove that $E_1 \cup E_2$ is also a connected if $E_1 \cap E_2 = \emptyset$.

(2) State and prove the theorem of nested intervals.

(3) Prove that continuous image of a connected set is also connected in a metric space.

(4) If $L\{f(t)\} = \bar{f}(s)$ then prove that

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\left[\bar{f}(s)\right], \text{ where } n = 1, 2, 3.......$$

(5) State convolution theorem and applying it find the inverse Laplace transform of $\frac{s}{(s^2 + a^2)^2}$.

3 (a) Answer any **Three** : **6**

(1) If $\emptyset : (G, *) -> (G', \Delta)$ be a homomorphism of groups then show that $\emptyset(a^{-1}) = [\emptyset(a)]^{-1}, \forall a \in G$.

(2) Let R be a ring and a, b $\in$ R then prove that

(i) a0=0a=0

(ii) a(–b)= –(a b) = (–a)b

(3) Give an example of a left ideal of $M_2(\mathbb{Z})$ which is not an ideal of $M_2(\mathbb{Z})$.

(4) Prove that the union of two ideals of a ring R if one of them is a superset of other.

(5) Show that the ring $\mathbb{Z}$ is a principal ideal ring.

(6) Justify: deg (f) $\leq$ deg(fg) for any two polynomials f & g of D[x].

(b) Answer any **Three** : **9**

(1) If $\varnothing:(G,*)->(G',\Delta)$ be a homomorphism of groups then show that $\varnothing^{-1}$(N') is a normal subgroup of G if N' is a normal subgroup of G'.

(2) Let I be an ideal of a ring R with unity. Then, prove that $I = \text{R}$ if $1 \in I$.

(3) Prove that a field has no proper ideal.

(4) Find deg(f g) if f=(1,2,5,0,0,...) & g=(1,2,5,7,0,0,...).

(5) Prove that the product of two monic polynomials is also a monic polynomial.

(6) State and prove the factor theorem for polynomials

(c) Answer any **Two** : **10**

(1) State and prove the first fundamental theorem of homomorphism of groups.

(2) Prove that a commutative ring R with unity is a field if it has no proper ideal.

(3) State and prove the remainder theorem for polynomials.

(4) State and prove the division algorithm for polynomials.

(5) Find the g.c.d. of the polynomials

$f(x) = x^3 + 3x^2 + 3x + 3$ & $g(x) = 4x^3 + 2x^2 + 2x + 2$ of

$\mathbb{Z}_5[x]$ and express it in the form $a(x)f(x) + b(x)g(x)$.
